technische universität dortmund

Dept. of Computer Science
Chair of Algorithm Engineering

Tutorial for

Introduction to Computational Intelligence in Winter 2015/16

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Lecture website: `https://tinyurl.com/CI-WS2015-16`

Sheet 4, Block II

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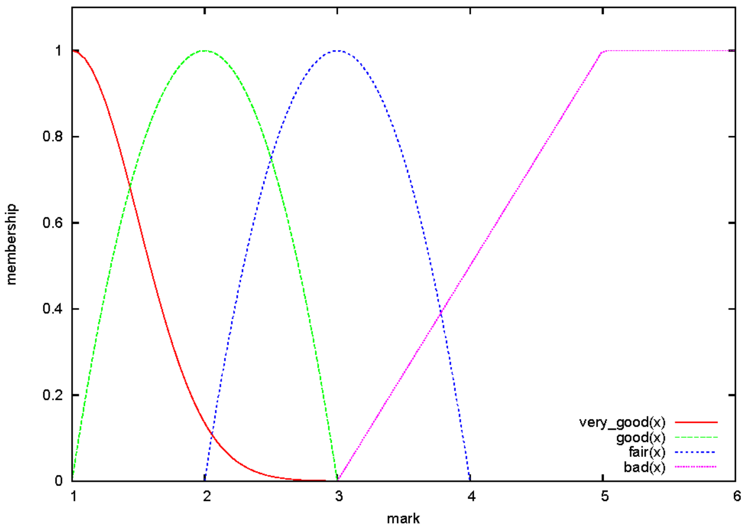
Due date: 13 January 2016, 2pm
Discussion: 14/15 January 2016

Exercise 4.1: Fuzzy Inference (5 Points)

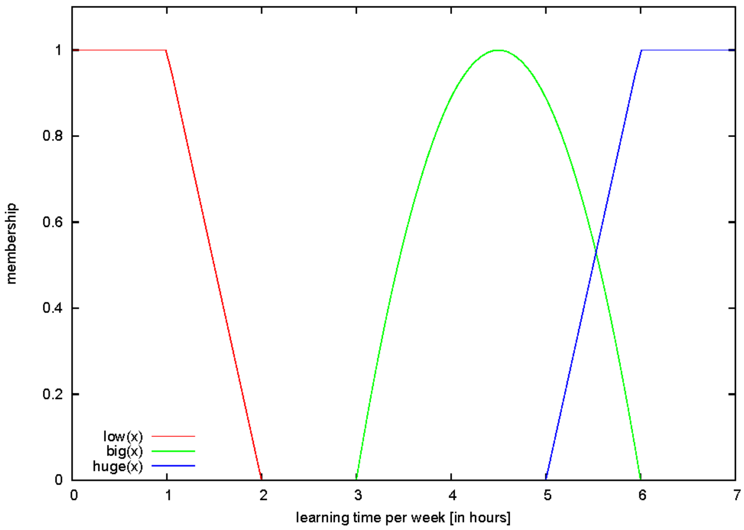
Consider the membership functions for the linguistic terms of the linguistic variable `mark`. Notice that outside the given range their values are zero!

$$
\begin{aligned}
\texttt{very\_good}(x) &= \exp(-2\,(x-1)^2) \text{ for } x \geq 1 \\
\texttt{good}(x) &= -(x-1)\,(x-3) \text{ for } x \in (1,3) \\
\texttt{fair}(x) &= -(x-2)\,(x-4) \text{ for } x \in (2,4) \\
\texttt{bad}(x) &= \min\{1, \frac{1}{2}(x-3)\} \text{ for } x > 3
\end{aligned}
$$



Below you can find the membership functions for the linguistic terms of the linguistic variable `learning_time`. Again, outside the given range their values are zero!

$$
\begin{aligned}
\texttt{huge}(x) &= \min\{x-5, 1\} \text{ for } x \geq 5 \\
\texttt{big}(x) &= -\frac{4}{9}(x-3)\,(x-6) \text{ for } x \in (3,6) \\
\texttt{low}(x) &= \min\{2-x, 1\} \text{ for } x < 2
\end{aligned}
$$



Consider the fuzzy proposition

if `learning_time` is `big` then `mark` is `good`,

and the given fuzzy fact

`mark` is `fair`.

Deduce the resulting fuzzy set over learning time using the Łukaciewicz implication $\mathrm{Imp}(a,b) = \min\{1, 1-a+b\}$ and the max-prod composition.

Sketch the membership function. Hint: Discretize the function and use a table of values.

Lösung
Fuzzy proposition: if `learning_time` is `big` then `mark` is `good`
fuzzy set for big learning time: $A(a) = -\frac{4}{9}(a-3)(a-6)$ for $a \in (3,6)$
fuzzy set for good mark: $B(b) = -(b-1)(b-3)$ for $b \in (1,3)$
fuzzy fact 'mark is fair': $B'(b) = -(b-2)(b-4)$ for $b \in (2,4)$

Value table for A, B, B':

a	3	4	5	6
A(a)	0	8/9	8/9	0

b	1	2	3
B(b)	0	1	0

b	1	2	3
B'(b)	0	0	1

$R(a,b) = Imp(A(a), B(b)) = \min\{1, 1 - A(a) + B(b)\}$

R	a=3	a=4	a=5	a=6
b=1	1	1/9	1/9	1
b=2	1	1	1	1
b=3	1	1/9	1/9	1

Generalized Modus tollens (GMT) using max-prod composition:

$$B' \circ R^{-1} = A' = (0\ 0\ 1) \circ \begin{pmatrix} 1 & 1/9 & 1/9 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1/9 & 1/9 & 1 \end{pmatrix} = (1\ 1/9\ 1/9\ 1)$$

max-prod: $(P \bullet Q)(x,z) = \max_{y \in Y}\{P(x,y) \cdot Q(y,z)\}$, P, Q fuzzy relations
$(B' \bullet R^{-1})(1,a) = \max_b\{B'(b) \cdot R^{-1}(b,a)\}$

e.g $(B' \bullet R^{-1})(1,4) = \max_b\{0 \cdot 1/9,\ 0 \cdot 1,\ 1 \cdot 1/9\} = 1/9$

Exercise 4.2: Fuzzy Implication (5 Points)

a) Use the increasing generator $g(x) = \sqrt{x}$ to derive a fuzzy implication. Does the resulting implication fulfill the axiom of contraposition? You are not allowed to use the theorem on slide 33 in lecture 7.

b) Check for all fuzzy implications below if they fulfill the axiom of contraposition, again without using the theorem (lec 7, slide 33):

- Reichenbach $\mathrm{Imp}(a, b) = 1 - a + a\,b$
- Łukaciewicz $\mathrm{Imp}(a, b) = \min\{1, 1 - a + b\}$
- Gödel $\mathrm{Imp}(a, b) = \begin{cases} 1 & a \leq b \\ b & \text{otherwise} \end{cases}$

Lösung
(a)
Definition of increasing generator (see lecture 6, slide 11).
$g(0) = 0$, strictly monotone increasing, $\forall a \in [0, 1] : c(a) = g^{-1}(g(1) - g(a))$

Given increasing generator: $g(x) = \sqrt{x} \quad \Rightarrow \quad g^{-1}(x) = x^2$

According to theorem from lecture (lec 7, slide 33), an increasing generator g with the implication $Imp(a, b) = g^{-1}(g(1) - g(a) + g(b))$ fulfills all 9 axioms for fuzzy implication (lec 7, slide 32). Solution completed.

More Details:
General proof of contraposition: $Imp(a, b) = Imp(c(b), c(a))$

$$\begin{aligned} Imp(c(b), c(a)) &= g^{-1}\Big(g(1) - g(c(b)) + g(c(a))\Big) \\ &= g^{-1}\Big(g(1) - g(g^{-1}(g(1) - g(b))) + g(g^{-1}(g(1) - g(a)))\Big) \\ &= g^{-1}\Big(g(1) - (g(1) - g(b)) + (g(1) - g(a))\Big) \\ &= g^{-1}(g(1) - g(a) + g(b)) \\ &= Imp(a, b) \end{aligned}$$

Proof of contraposition for specific generator g:

$c(a) = g^{-1}(g(1) - g(a)) = (\sqrt{1} - \sqrt{a})^2 = (1 - \sqrt{a})^2$
$Imp(a, b) == g^{-1}(g(1) - g(a) + g(b)) = g^{-1}(\sqrt{1} - \sqrt{a} + \sqrt{b}) = (1 - \sqrt{a} + \sqrt{b})^2$

$$\begin{aligned} Imp(c(b), c(a)) &= g^{-1}(g(1) - g(c(b)) + g(c(a))) \\ &= \left(\sqrt{1} - \sqrt{(1 - \sqrt{b})^2} + \sqrt{(1 - \sqrt{a})^2}\right)^2 \\ &= (1 - (1 - \sqrt{b}) + (1 - \sqrt{a}))^2 \\ &= (1 - (1 - \sqrt{b}) + (1 - \sqrt{a}))^2 \\ &= (1 - \sqrt{a} + \sqrt{b})^2 \\ &= Imp(a, b) \end{aligned}$$

(b)
Check axiom of contraposition with standard complement $c(a) = 1 - a$.

Reichenbach:
$Imp(c(b), c(a)) = 1 - (1 - b) + (1 - b)(1 - a) = 1 - 1 + b + 1 - b - a + ab = 1 - a + ab = Imp(a, b)$

Łukaciewicz:
$Imp(c(b), c(a)) = \min\{1, 1-(1-b)+(1-a)\} = \min\{1, 1-1+b+1-a\} = \min\{1, 1-a+b\} = Imp(a, b)$

Gödel:

$$Imp(c(b), c(a)) = \begin{cases} 1 & (1-b) \leq (1-a) \\ (1-a) & \text{otherwise} \end{cases} = \begin{cases} 1 & a \leq b \\ 1-a & \text{otherwise} \end{cases} \neq \begin{cases} 1 & a \leq b \\ b & \text{otherwise} \end{cases} = Imp(a, b)$$

Exercise 4.3: Fuzzy Controller (10 Points)

Script trainsim.R contains a a simple simulator for a train. The function trainspeed(currentSpeed, force) returns the speed on the following time step. The new speed is calculated based on the speed at the time and the applied force: negative values mean braking, positive values acceleration.

The value of the acceleration/braking force can be in the interval $[-200, 25]$. In combination with the other parameters, this means that the maximum speed of the train is 42.8 m/s ($\approx$ 154 km/h) and the braking distance is about 538 m. The traveled distance in meter can be obtained by simply summing the returned speeds, because the unit is m/s and we use time steps of one second in the simulation.

Your task is to implement a Mamdani controller for the speed of a train in R that helps the train to travel a given distance as precisely and fast as possible (imagine that the train tracks end in a terminal station).

a) Using your own perception, define membership functions for the linguistic variables *speed*, *remaining distance*, and *driving/braking force*.

b) Define a comprehensible fuzzy rule system for the controller.

c) Implement the control loop for the Mamdani controller using the center of gravity method for defuzzyfication.

d) Comment your sourcecode.

e) Start the train for a desired travel distance of 1000 m and 10 km with initial speeds of 0, 20, and 42.8 m/s. Report for each of the six settings the traveled distance and the required time. Plot the train's speed over time.

Lösung

Mamdani Controller (lec 8, slide 17):
max-aggregation, $R(x, y) = \min(a, b)$, center of gravity method for defuzzyfication